

# Mathematics Curriculum

## Scope and Sequence, Kindergarten through Gr. 8

### Overview: Mission and Core Competency Categories

The Math Program at The School at Columbia University actively cultivates confident mathematicians through a rigorous, coherent curriculum grounded in collaboration, joyful inquiry, responsive differentiation, cultural relevance, and equitable access for all learners. Students engage deeply with mathematics through a variety of meaningful applications embedded in interdisciplinary and real-world contexts. Core competency categories are:

**Numeracy, Place Value, and Computation:** This category builds the foundation for all mathematical understanding by developing number sense, fluency with counting, and understanding of place value. Students learn how numbers are composed and decomposed, compare and order numbers, and use place value to perform operations efficiently. Computation includes developing accurate, flexible, and efficient strategies for addition, subtraction, multiplication, and division, grounded in a conceptual understanding of the base-ten system.

**Operations and Algebraic Thinking:** Students develop an understanding of the properties of operations and the relationships among them. They represent and solve problems involving the four operations, explore patterns and relationships, and begin to generalize arithmetic thinking into early algebraic reasoning. This category also includes understanding the meaning of the equal sign, building toward writing and solving equations and expressions.

**Measurement and Data Analysis:** In this category, students measure and estimate lengths, time, volume, and weight using appropriate tools and units. They learn to collect, organize, and interpret data through graphs and charts. This category emphasizes reasoning about units, understanding measurement as a process of iteration, and using data to make informed decisions, spot trends, and draw conclusions.

**Geometry:** Students explore the properties and relationships of shapes and spatial structures. They identify, describe, compare, and classify shapes based on attributes, and begin reasoning about lines, angles, symmetry, and transformations. Geometry also supports spatial reasoning and connects to measurement concepts such as area, perimeter, and volume as well as early fraction concepts.

**Fractions and Decimals:** This category introduces students to rational numbers as extensions of the number system. Students develop a deep understanding of fractions as numbers and parts of a whole, as well as as operators, measures, and quotients. They learn to compare, represent, and perform operations with fractions and decimals, building fluency with equivalent representations and reasoning about size, equivalence, and computation strategies.

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## KINDERGARTEN: Core Competencies

### Numeracy, Place Value, and Computation

- Count forward by 1s and 10s to 100 and backward from 20.
- Recognize, write, and represent numbers from 0 to 20.
- Compare numbers up to 20 using terms like more, less, and equal.
- Develop a sense of quantity through subitizing (instantly recognizing small quantities).
- Understand teen numbers (11-19) as a group of ten ones and some more ones.

### Operations and Algebraic Thinking

- Use objects and drawings to model and solve addition and subtraction problems within 10.
  - Fluently add and subtract within 5.
- Understand that numbers represent quantities and can be broken apart and put together in different ways (e.g.,  $5 = 2 + 3 = 4 + 1$ ).
- Explore the relationship between addition and subtraction.
- Recognize and extend simple patterns (AB, AAB, etc.).

### Measurement and Data Analysis

- Compare objects using measurable attributes (length, height, weight).
- Sort objects by color, size, shape, or other attributes. Count the number of objects in each category.

### Geometry

- Describe spatial relationships using relative position words like above, below, next to, and between.
- Explore two- and three-dimensional objects and describe their similarities, differences, and other attributes using informal language.

## GRADE 1: Core Competencies

### Numeracy, Place Value, and Computation

- Count forward and backward to 120 starting from any number; relate counting to addition and subtraction.
- Read and write numerals to 120 and represent numbers with objects and drawings.
- Understand place value of two-digit numbers (tens and ones).
- Compare two-digit numbers using symbols.
- Use place value understanding and properties of operations to add and subtract, including:
  - adding a two-digit number to a one-digit number and to a multiple of 10;
  - mentally finding 10 more or 10 less than a given number; and
  - subtracting multiples of 10 from multiples of 10 (within 10–90).

### Operations and Algebraic Thinking

- Use addition and subtraction to solve one-step word problems within 20 involving adding to, taking from, putting together, taking apart, and/or comparing, with unknowns in all positions.
- Understand the relationships between counting, addition and subtraction.
- Apply properties of operations and strategies to add and subtract within 20.
- Fluently add and subtract within 10.
- Reason about equations with unknowns in all positions and whether an equation is true or false.
- Use manipulatives, models and place value strategies to solve, show work, and explain thinking for addition and subtraction.



## GRADE 1: Core Competencies

### Measurement and Data Analysis

- Develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.
- Tell and write time to the hour and half-hour on analog and digital clocks.
- Organize, represent, and interpret data using simple charts and graphs with up to 3 categories.

### Geometry

- Identify, describe, compare, compose, and draw 2D and 3D shapes using defining and non-defining attributes.

### Fractions and Decimals

- Represent and describe equal shares of circles and rectangles as halves and quarters/fourths.



## GRADE 2: Core Competencies

### Numeracy, Place Value, and Computation

- Understand that three-digit numbers are made of hundreds, tens, and ones.
- Skip count by 2s, 5s, 10s, and 100s.
- Compare and order numbers up to 1,000 using symbols and place value knowledge.
- Add and subtract within 1,000 using strategies based on place value and properties of operations.
- Fluently add and subtract within 100.
- Mentally add and subtract within 20.

### Operations and Algebraic Thinking

- Solve one and two-step word problems using addition and subtraction, involving adding to, taking from, putting together, taking apart, and/or comparing, with unknowns in all positions.
- Begin working with equal groups as a foundation for multiplication.
- Represent repeated addition with arrays.
- Generalize about even and odd numbers.

### Measurement and Data Analysis

- Measure length using whole standard units (inches, centimeters).
- Estimate and compare lengths and solve addition and subtraction word problems involving length.
- Tell and write time to the nearest five minutes; understand a.m. and p.m.
- Work with money: identify coins, count collections, and solve simple word problems.
- Organize, represent and interpret data with single-unit scale bar graphs and picture graphs with up to 4 categories; use graphs to solve put-together, take-apart, and compare problems.

### Geometry

- Recognize and draw shapes with specific attributes (e.g., sides, vertices, angles).

### Fractions and Decimals

- Partition and describe equal halves, thirds, and fourths of circles and rectangles, even when the parts differ in appearance.

## GRADE 3: Core Competencies

### Numeracy, Place Value, and Computation

- Understand place value up to 1,000 and round numbers to the nearest 10 or 100.
- Read, write, and compare numbers using base-ten understanding.
- Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction
- Multiply multiples of 10 by a single-digit number.

### Operations and Algebraic Thinking

- Multiply and divide within 100 using strategies and understanding of the relationship between operations.
- Fluently compute all multiplication and division facts through  $10 \times 10$  by the end of the year, building on strengths with 2s, 5s, and 10s.
- Solve two-step word problems using all four operations.
- Identify and work with arithmetic patterns.

## Measurement and Data Analysis

- Solve problems involving measurement and estimation of time, liquid volume, and mass.
  - Read measurements for volume, length, and mass using standard units.
  - Measure lengths to nearest  $\frac{1}{4}$  of an inch and millimeter.
  - Tell and write time to the nearest minute and solve elapsed time problems.
- Recognize area as the amount of same-size unit squares required to cover a shape without gaps or overlaps and perimeter as the total length around a figure.
  - Solve real-world and mathematical problems involving area and perimeter of rectilinear figures.
  - Relate area and perimeter to the operations and properties of multiplication and addition.
- Organize, represent and interpret data using line plots, scaled bar graphs, and scaled picture graphs.

## GRADE 3: Core Competencies

### Geometry

- Classify shapes into the categories of triangles, quadrilaterals, pentagons, and hexagon based on the number of sides and vertices.
  - Classify quadrilaterals and triangles into sub-categories using precise attributes such as number of equal and/or parallel sides and presence of right angles.
  - Generalize that a shape can have more than one name depending on its attributes.

### Fractions and Decimals

- Understand fractions as a part of a whole and a number on the number line.
- Represent, interpret, and compare fractions using geometric models, linear models (including number lines and tape diagrams), and set models in concrete, pictorial, and abstract forms.
- Generate equivalent fractions and compare fractions with the same numerator or denominator with geometric models and number lines.



## GRADE 4: Core Competencies

### Numeracy, Place Value, and Computation

- Generalize place value understandings to 1,000,000,000 and use them to round and compare multi-digit numbers.
- Read, write, and represent numbers in standard, expanded, and word form.
- Add and subtract multi-digit numbers fluently using standard algorithms.
- Multiply multi-digit numbers using strategies based on place value and operational properties, such as area models, partial products, ratio tables, decomposition, and adjusting.

### Operations and Algebraic Thinking

- Divide up to four-digit numbers by one-digit divisors using strategies based on place value, such as area models, partial quotients, and ratio tables.
- Develop familiarity with factors and multiples.
- Solve multi-step word problems with all four operations.
  - Interpret multiplication as a comparison between quantities.
  - Interpret remainders in problem solving contexts.

### Measurement and Data Analysis

- Solve problems involving measurement and conversion of larger units to smaller units within the same system (e.g., feet to inches, meters to centimeters, etc).
- Determine volume of rectangular prisms by modeling volume as iterating layers of area (base  $\times$  height), connecting the filling of 3-dimensional space with unit cubes to the formula  $V = l \times w \times h$  and solving real-world problems using metric and customary cubic units.
- Measure angles with a protractor and solve problems involving unknown angles.

## GRADE 4: Core Competencies

### Geometry

- Classify polygons based on precise geometric attributes such as parallel lines, perpendicular lines, side lengths, symmetry, and types of angles.
- Graph points in the first quadrant of the coordinate plane to solve real-world and mathematical problems.

### Fractions and Decimals

- Recognize, model, and generate equivalent fractions.
- Compare fractions with unlike denominators.
- Add and subtract fractions and mixed numbers with like denominators.
- Multiply a fraction by a whole number.
- Express numbers in decimal notation for tenths and hundredths and compare decimals.



## GRADE 5: Core Competencies

### Numeracy, Place Value, and Computation

- Recall multiplication facts (1–12) and division facts (up to 144) with fluency.
- Fluently employ the standard multiplication algorithm for multi-digit numbers.
- Compute multi-digit division using the standard division algorithm.
- Use exponents to express whole numbers in exponential form and understand what the exponent and base represent.
- Perform prime factorization and express prime factors using exponent notation.

### Operations and Algebraic Thinking

- Build on multiplication strategies by connecting visual models (e.g., arrays and area models) to the standard multiplication algorithm for multi-digit numbers.
- Understand division as fractions, scaling, sharing or grouping and recognize situations that involve division. Solve division problems, including in real-world contexts, using a variety of strategies and visual models.
- Explore number theory concepts such as identifying and classifying prime, composite, square numbers, factors, and multiples.

### Measurement and Data Analysis

- Apply knowledge of measurement conversions within the same system (e.g., inches to feet, milliliters to liters) to solve real-world problems.
- Discover the relationship between fractions, decimals, and percentages and probability
- Delineate the difference between theoretical and experimental probability, and understand how to calculate both given a certain scenario.
- Understand the probability of a chance event and develop probability models for simple events.

## GRADE 5: Core Competencies

### Geometry

- Model multiplication and division through applications of area and perimeter.
- Use precise angle vocabulary (acute, right, obtuse) to identify and classify triangles (acute, right, isosceles, etc.).
- Understand that the sum of angles in a triangle is always 180 degrees.
- Use tools and visual reasoning to construct geometric figures and explore attributes of shapes.
- Plot points and figures on the coordinate plane.

### Fractions and Decimals

- Understand that division can be expressed as a fraction:  $a \div b = \frac{a}{b}$ , and apply this to reasoning about fractional quantities.
- Extend fraction representations on a number line to include more landmark fractions, decimals, and percents.
- Apply fraction equivalence understanding to find common denominators
- Use flexibly and fluidly various fraction models like pie charts, bar models, clocks, ratio tables, and number lines
- Convert between mixed numbers and improper fractions.
- Add and subtract fractions with like and unlike denominators with both models and equations.
- Understand multiplication of fractions as scaling (resizing).
- Multiply fractions by fractions using visual models to support conceptual understanding.
- Divide a unit fraction by a whole number and divide a whole number by a unit fraction, using models to build meaning.
- Move flexibly between fractions, decimals, and percent representations.
- Add and subtract decimals to the hundredths place with accuracy and understanding.
- Multiply decimals using place value understanding and models.
- Divide decimals by whole numbers using models and reasoning.

## GRADE 6: Overview & Core Competencies

In sixth grade, students strengthen their foundation in number theory, fractions, decimals, algebra, geometry, and statistics through problem-solving and real-world applications. They develop fluency with operations involving fractions and decimals, explore prime factorization, and apply concepts like greatest common factor (GCF) and least common multiple (LCM) to solve problems efficiently. Students begin writing and evaluating expressions, solving one-step equations, and working with negative numbers and rational values. In the second semester, students analyze data using statistical tools, explore geometry through hands-on projects, and apply formulas for surface area and volume. Collaborative, interdisciplinary projects deepen understanding and allow students to engage with advanced mathematical thinking in creative ways.

### Numeracy, Place Value, and Computation

- Understand and identify multiplication and division scenarios and solve problems using a variety of strategies and algorithms.
- Extend understanding of the number line to include negative numbers and opposites.
- Perform all four operations with integers.
- Apply the order of operations to evaluate numerical and algebraic expressions accurately.
- Classify numbers within the real number system and work fluently with negative numbers, including understanding absolute value and computing with all four operations.

### Operations and Algebraic Thinking

- Understand and use absolute value to describe distance from zero.
- Use number theory concepts to explore relationships between products, factors, and divisors. This includes finding the greatest common factor (GCF), least common multiple (LCM), and prime factorization, and expressing prime factors using factor strings and exponential form.
- Solve real-world problems using proportions, including solving using cross multiplication.
- Understand the relationships between fractions, decimals, and percents, and solve percent problems such as finding a percent of a number, percent increase/decrease, and missing values in percent equations.

## GRADE 6: Core Competencies

### Measurement and Data Analysis

- Create and interpret different types of graphs including bar graphs, line graphs, and dot plots to represent and analyze data.
- Begin to make sense of data and its representation in real-world contexts.

### Geometry

- Calculate the perimeter of various shapes, including more complex or irregular figures.
- Understand the concept of area and apply area formulas to calculate the area of squares, rectangles, triangles, parallelograms, trapezoids, circles, and compound figures.
- Distinguish between area and perimeter in mathematical and real-world contexts.
- Explore and calculate the volume of cubes, cuboids, prisms, and cylinders using volume formulas.
- Find distances between points that lie on the same vertical or horizontal line.
- Identify and classify 2D shapes and 3D solids by their attributes and nets, using precise geometric vocabulary.
- Use tools and reasoning to measure angles and sides of polygons and decompose complex figures into simpler shapes.
- Determine surface area using nets and apply this to cubes, cuboids, and prisms.
- Plot and interpret points in all four quadrants of the coordinate plane using appropriate vocabulary (origin, axes, quadrants, coordinates, etc.).

## GRADE 6: Core Competencies

### Fractions and Decimals

- Apply understanding of equivalence to represent fractions in simplest form.
- Multiply and divide fractions and mixed numbers, including real-world applications.
- Solve multi-step problems using all fraction operations with appropriate models and equations.
- Exhibit fluency in adding, subtracting, and multiplying decimals using both models and standard algorithms, and apply these skills to solve problems in real-world contexts.
- Divide decimals by whole numbers and by other decimals with conceptual understanding and precision.
- Understand and use ratios and unit rates to describe relationships between quantities.
- Solve unit rate problems, including those involving fractions (e.g., finding price per item, constant speed).
- Represent and interpret ratio relationships with tables, double number lines, tape diagrams, and graphs.
- Understand and use percent as a rate per 100; represent a part of a whole as a percent.
- Solve problems involving percent, including finding the whole, part, or percent in real-world contexts.
- Connect equivalent ratios and fractions to develop proportional reasoning.
- Recognize when two quantities are in a proportional relationship, and represent proportional relationships with equations, tables, and graphs.
- Use fraction equivalence and proportional reasoning to solve problems involving equivalent ratios and rates.

### Algebraic Expressions and Equations

- Build foundational algebra skills by defining and using variables.
- Substitute values into expressions and solve one-step and two-step linear equations.
- Write and interpret equations from real-life scenarios to model mathematical relationships.

## GRADE 7: Overview & Core Competencies

Seventh grade mathematics expands students' understanding of rational numbers and introduces them to algebraic thinking in greater depth. Students apply operations with rational numbers in increasingly complex scenarios, and build fluency with expressions, equations, and proportional reasoning. Emphasis is placed on developing confidence with pre-algebraic skills that lay the groundwork for future success in Algebra 1. Topics introduced early in the year are revisited and extended, allowing students to refine their understanding through practice and application. Differentiation is supported through challenge tasks, math projects, and enrichment opportunities that promote curiosity and critical thinking.

### Numeracy, Place Value, and Computation

- Recognize and approximate square roots of perfect and non-perfect squares on a number line.
- Write, simplify, and solve expressions using exponent rules and properties, including simple exponential equations.
- Simplify radical expressions, including those with variables, and solve radical equations.
- Work with powers of 10 and represent numbers using scientific notation.

### Operations and Algebraic Thinking

- Apply strategies to find the greatest common factor (GCF) and least common multiple (LCM), in algebraic expressions.

## GRADE 7: Core Competencies

### Measurement and Data Analysis

- Use set theory to describe and compare groups using concepts like union and intersection, and understand set-builder notation.

### Geometry

- Use geometric vocabulary to describe and name lines, segments, and angles (e.g.,  $\angle ABC$ , vertical angles, parallel line relationships - alternate interior/exterior and corresponding angles), and understand properties of triangles, triangle angle sum theorem, and relationships between angles.
- Explore concepts of similarity, congruence, symmetry, and transformations.
- Plot and interpret points in all four quadrants of the coordinate plane using appropriate vocabulary (origin, axes, quadrants, coordinates, etc.).

### Algebraic Expressions and Equations

- Understand and define variables and substitute values into linear expressions.
- Solve one-step linear equations and apply these strategies for solving multi-step equations.
- Represent linear relationships on a coordinate plane, including identifying and interpreting slope.
- Write, graph, and interpret linear equations in slope-intercept form.

## GRADE 8: Overview & Core Competencies

Eighth grade students take Algebra 1, a high school-level course focused on building deep understanding of algebraic reasoning through real-world contexts, data analysis, and mathematical modeling. The curriculum emphasizes conceptual understanding and procedural fluency, while encouraging students to represent mathematical relationships in multiple ways—algebraically, graphically, numerically, and verbally. Students engage in a carefully sequenced progression of topics including polynomials, linear and exponential functions, systems of equations, statistics, and quadratics. Throughout the course, students develop problem-solving strategies, use technology to analyze data, and apply algebra to authentic scenarios, preparing them for future success in higher-level mathematics.

### Operations and Algebraic Thinking

- Understand and apply the commutative, associative, and distributive properties in algebraic contexts.

### Measurement and Data Analysis

- Create, interpret, and analyze data using box and whisker plots, frequency tables, and scatter plots.
- Calculate and analyze measures of center (mean, median, mode) and variability (standard deviation, outliers).
- Analyze and compare data sets using measures of center and variability, as well as graphical representations and additional statistical information.
- Determine the correlation coefficient and line of best fit to analyze relationships between data sets.

### Geometry

- Use the Pythagorean Theorem to find missing side lengths in right triangles.

## GRADE 8: Core Competencies

### Algebraic Expressions and Equations

- Substitute values into non-linear expressions, simplify polynomials, and factor polynomial expressions.
- Simplify and perform operations with rational expressions.
- Solve and graph inequalities in one and two variables.
- Solve systems of equations and inequalities using graphing, substitution, and elimination, and interpret their meanings in context.

### Functions

- Understand the concept of a function, use function notation, and interpret function graphs in real-world contexts.
- Graph linear functions using various methods, including t-charts, slope-intercept, standard form, and point-slope forms.
- Graph absolute value functions using t-charts and the vertex formula.
- Graph exponential functions using t-charts, vertex, and standard form.
- Solve quadratic equations using graphing (in vertex and standard form), factoring, completing the square, and the quadratic formula.
- Interpret the meaning of solutions as the zeros of a function and apply function models to real-world problems.

